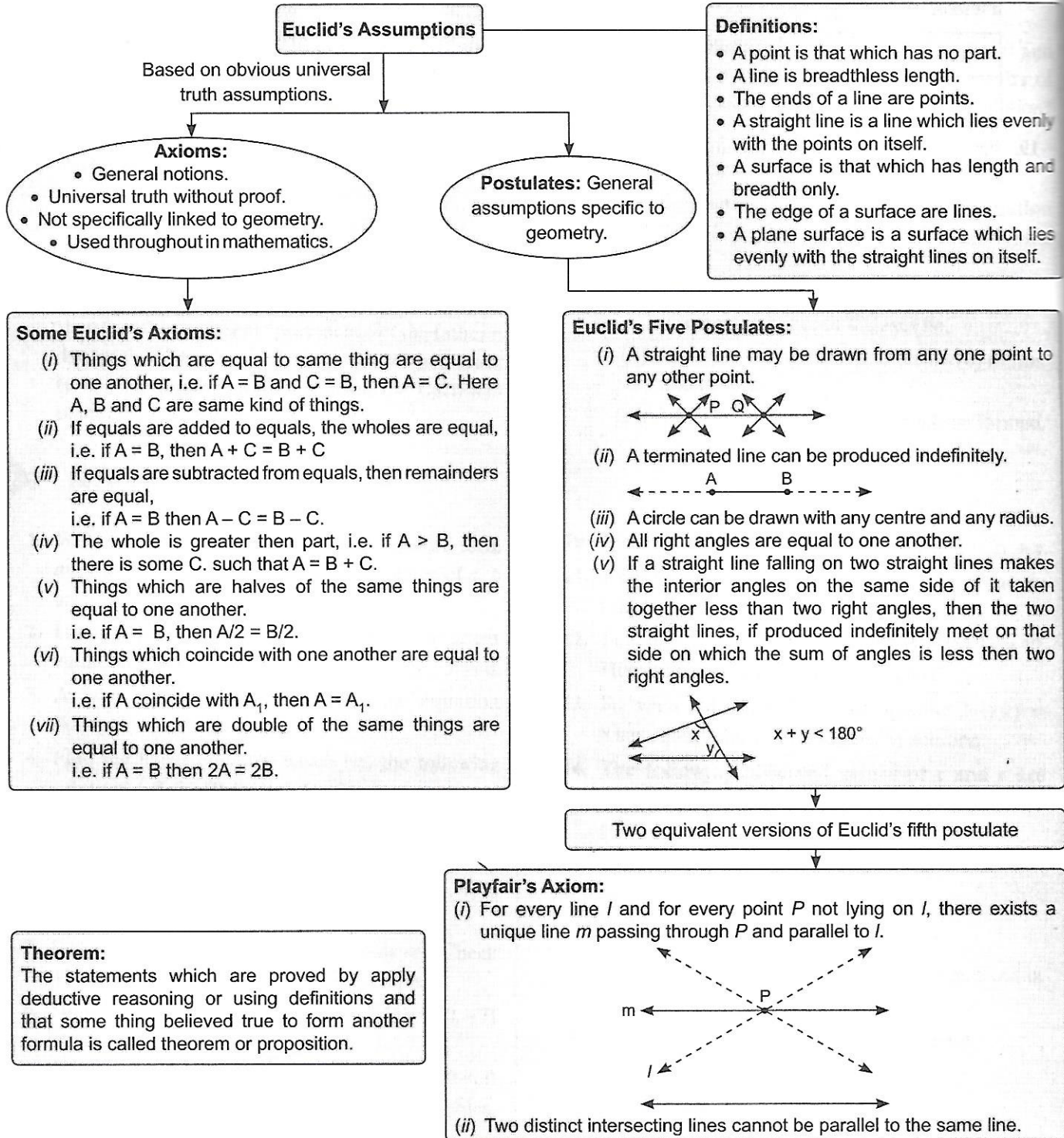


5

INTRODUCTION TO EUCLID'S GEOMETRY

Euclid's Definitions, Axioms and Postulates



➤ SOLVED QUESTIONS BASED ON EXERCISES 5.1 AND 5.2

Very Short Answer Type Questions [1 Mark]

1. What is the difference between 'axiom' and postulate?

Sol. Axioms are the assumptions that are used throughout the mathematics and not specifically linked to geometry, while postulate are the assumptions that are specific to geometry.

2. Area of rectangle is equal to area of triangle and area of triangle is 25 cm^2 . Find the area of square whose area is equal to area of rectangle.

Sol. Since the things which are equal to the same thing are equal to one another. So, the area of square 25 cm^2 .

3. A line cannot be added to a rectangle nor can an angle be compared with the hexagon. What do you conclude?

Sol. Magnitude of same kind can be compared and added but magnitude of different kinds cannot be compared.

4. Every thing equal itself. What does it justify?

Sol. If two things are same, then they are equal. It is the justification of principle of superposition.

5. How many common points do two distinct lines have?

Sol. Two distinct lines cannot have more than one point in common.

6. For every line l and a point P not lying on it, find the number of lines that passes through P and parallel to l .

Sol. Only one line is there that passes through point P and parallel to l .

7. Let $\angle A = \angle B$ and $\angle B = \angle C$. State the Euclid's axiom according to which relation between $\angle A$ and $\angle C$ is established?

Sol. Euclid's axiom is "Things which are equal to same thing are equal to one another."

Short Answer Type Questions I [2 Marks]

8. Which of the following statements are true and which are false? Justify you answer.

(a) If two circles are equal, than their radii are unequal.

(b) Given two distinct points, there is a unique line that passes through them.

Sol. (a) **False:** If two circles superimpose exactly on one another, then they coincide. So their centres and region bounded by their boundaries must also coincide. Therefore, their radii will also be coincide. Hence equal circles have equal radii.

(b) **True:** Only one line can be passed through two distinct points.

9. It is known that $p + q = 6$ and $p = r$. Show that $r + q = 6$.

Sol. Given: $p + q = 6$ and $p = r$

So, from Euclid's axiom, that if equals are added to equals, the wholes are equal.

Therefore, we get

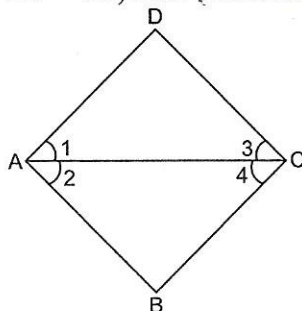
$$p + q = r + q$$

\Rightarrow

$$r + q = 6$$

Hence proved.

10. In the given figure, if $\angle 2 = \angle 4$ and $\angle 4 = \angle 1$, then prove that $\angle 1 = \angle 2$.



Sol. Given: $\angle 2 = \angle 4$ and $\angle 4 = \angle 1$, using Euclid's axiom, things which are equal to the same thing are equal to one another.

Hence,

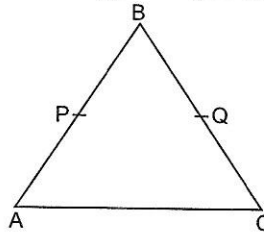
$$\angle 2 = \angle 4 = \angle 1$$

\Rightarrow

$$\angle 1 = \angle 2$$

Hence proved.

11. In the given figure, if $AB = BC$ and $AP = CQ$, then prove that $BP = BQ$.



Sol. Given:

$$AB = BC$$

...(i)

and

$$AP = CQ$$

...(ii)

According to Euclid's axiom, if equals are subtracted from equals, the remainders are equal.

Therefore, on subtracting (ii) from (i), we get

$$AB - AP = BC - CQ$$

(Given $AP = CQ$)

\Rightarrow

$$BP = BQ$$

Hence proved.

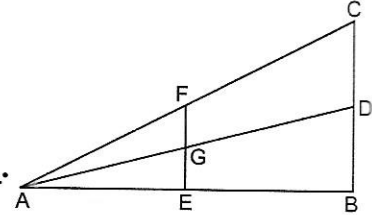
12. By using Euclid's axioms, complete the following statements with the name of straight line from the figure.

(a) If AE is subtracted from AB, then $AB - AE =$ _____.

(b) If GE is added to FG, then result is equal to _____.

(c) If GD is added to AG, the whole is equal to _____.

(d) If AF is subtracted from AC, the remainder is equal to _____.



Sol. (a) EB (b) FE (c) AD (d) CF

13. Solve the equation, $x - 10 = 25$ and state which axiom do you use here.

Sol. $x - 10 = 25$

On adding 10 both sides, we have

$$x - 10 + 10 = 25 + 10 \Rightarrow x = 35$$

Here, we use Euclid axiom, "If equal be added to the equal, the whole are equal."

14. Ram and Ravi have the same weight. If they each gain weight by 2 kg, how will there new weights be compared?

[NCERT Exemplar]

Sol. Let the weight of Ram and Ravi be x kg each. On gaining 2 kg weight, new weight of Ram and Ravi = $(x + 2)$ kg. But according to euclid's axiom, the wholes are equal. So, weight of Ram and Ravi are again equal.

15. Prove that every line segment has one and only one mid-point on it.

Sol. Let R is the mid-point of PQ, such that $PR = QR$.

To prove : PQ has one and only one mid-point R.

Proof : Let T be the mid-point of PQ other than R, then

$$PR = \frac{1}{2} PQ \Rightarrow PT = \frac{1}{2} PQ$$

Therefore,

$$PR = PT.$$

Which possible only when R and T coincide with each other. Hence proved.

16. In the given figure $PR = QS$, then show that $PQ = RS$. Name the mathematician whose postulate/axiom is used for the same.



[CBSE 2014, 15]

Sol.

$$PR = QS$$

(Given)

Subtraction QR both sides, we have

\Rightarrow

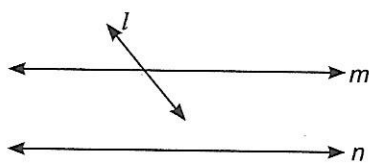
$$PR - QR = QS - QR \Rightarrow PQ = RS$$

We used here Euclid's axiom to prove the result which states that if equals are subtracted from equals, then remainders are equal.

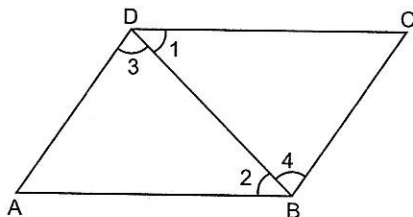
PRACTICE QUESTIONS BASED ON EXERCISES 5.1 AND 5.2

- State any one Euclid's axiom.
- According to Euclid's definition, what is a point?
- How many dimensions does solid have?
- Through two points, how many lines can be drawn?
- How many dimensions does a line have?
- If $\angle 1 = \angle 3$, $\angle 3 = \angle 4$ and $\angle 2 = \angle 4$. What is the relation between $\angle 1$ and $\angle 2$? Give reason for your answer.
- Show that the two lines which are both parallel to the same line, are parallel to each other.
- If $AB = 5$ cm $CD = 10$ cm and $PQ = 2AB$. Are CD and PQ equal? Which axiom is used for proving this?
- $AB = 4$ cm and length of PQ and CD is three-fourth the length of AB . Are PQ and CD equal?
- Which of the following statements are true and which are false?
 - The edge of the surfaces are points.
 - Two lines can have only one point in common.
 - Two distinct points A and B , there exists a third point C which is in between A and B .
 - Breadth and length, both are undefined term in the Euclid's axiom "A line is a breadth less length."

- n, l, m are three lines in the same plane shown in the given figure. If l intersect m and $n \parallel m$. Show that l also intersect n .

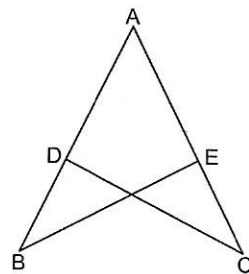


- In the given figure, it is given that $\angle 1 = \angle 4$ and $\angle 3 = \angle 2$. By which Euclid's axiom, it can be shown that if $\angle 2 = \angle 4$, then $\angle 1 = \angle 3$.



- X, Y and Z are the points lies on the boundary of a circle. If O is the centre of the circle, then why OX, OY and OZ are equal?
- Prove that a shorter line whose length is equal to given line can be cut off from the other longer straight line.

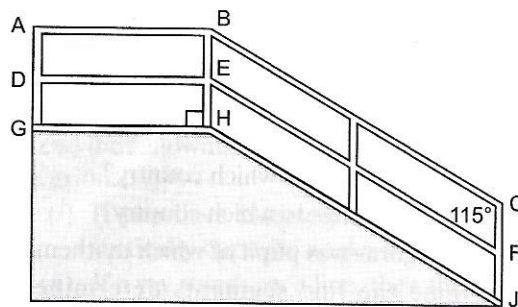
- In the given figure,
 $AB = AC$
 $AD = AE$
 Prove that $BE = DC$



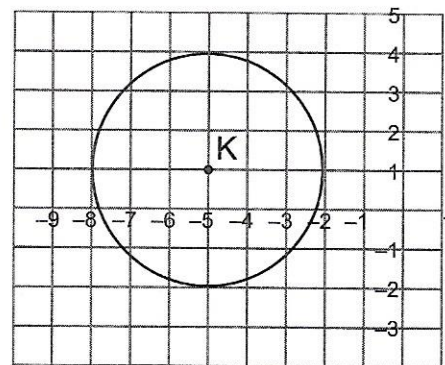
- Line AB, AC, AD and AE are parallel to line ' m '. What conclusion would you like to draw about the points A, B, C, D and E ?

Value Based Questions

- Monica wants to develop a circular ring of different flowers around a lamp post considering lamp post as centre. How many such circular strips she can create? Which Euclid's axiom is used to answer the above stated problem? She wrote a message "Do not pluck flowers". What value is she exhibiting by doing so? State one more axiom given by Euclid other than the axiom used above.
- Teacher held two sticks AB and CD of equal length in her hands and marked their mid-points M and N respectively. She then asked the students whether AM is equal to ND or not. Arpita answered yes. Is Arpita correct? State Euclid's axiom that support her answer. Which characteristics of Arpita you want to inculcate in your nature?
- In the given figure, ADG represents a College gate followed by a ramp HJ instead of stairs to make easy for physically challenged students to climb. HJ is equal to mid support EF and EF is equal to BC . What is the relation between BC and HJ ? State the Euclid's Axiom behind this fact. Which value college is exhibiting by placing ramps instead of stairs? Mention one more Euclid's axiom. [CBSE 2014]



4. A circular pond in a village is full of fishes. Rohan everyday feeds the fishes. What value is he exhibiting by doing so? With centre as K in the figure, how many circles can be drawn? State Euclid's axiom which supports this statement. Also, give two axioms of Euclids. [CBSE 2016]



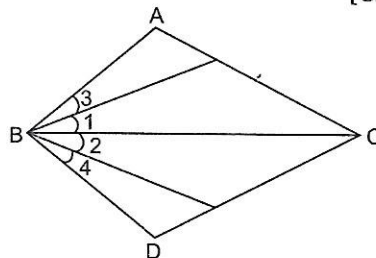
INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

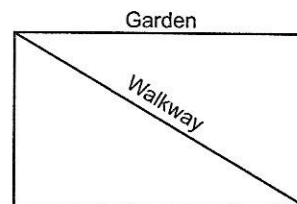
- List the three basic steps from solid to points in Euclid's geometry.
- How many dimensions does a surface have?
- How many dimensions does a point have?
- Euclid divided his famous treatise "The Elements" into how many chapters?
- What are total number of propositions in The Elements?
- What are boundaries of solids known as?
- What are boundaries of surfaces known as?
- In Indus Valley Civilisation (about 3000 B.C.), what is the ratio of dimensions in the bricks which were used for construction work.
- A pyramid is a solid figure, the base is of which shape?
- What are the side faces of a pyramid known as?
- It is known that if $x + y = 10$, then $x + y + z = 10 + z$. State the Euclid's axiom that illustrates the statement.
- In ancient India, name the shapes of altars that were used for household rituals.
- Find the number of interwoven isosceles triangles in Sriyantra (in the Atharvaveda).
- Greek's emphasised on deductive reasoning. If Yes, why?
- In ancient India, for which purpose altars with combination of shapes like rectangles, triangles and trapeziums were used.
- Euclid belongs to which country?
- Thales belongs to which country?
- Pythagoras was pupil of which mathematician?
- Name the line segments determined by three collinear points P, Q and R. [HOTS]

Short Answer Type Questions I [2 Marks]

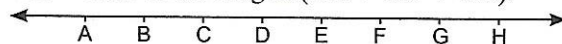
20. In the given figure, we have $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that $\angle ABC = \angle DBC$. State the Euclid's axiom used by you. [CBSE 2016]



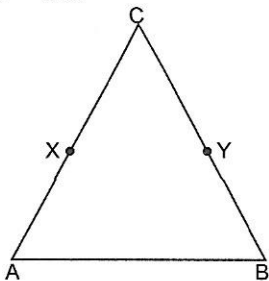
21. Mr. Kapoor is making a straight walkway in the garden joining the diagonally opposite points for the purpose of morning walk of elderly people (as shown in the figure). Which axiom states uniqueness of the line joining two points? State any two postulates of Euclid's. [CBSE 2014]



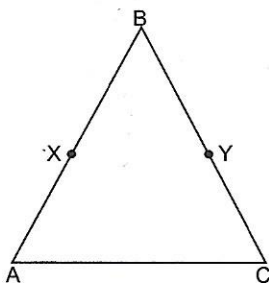
22. Attempts to prove Euclid's fifth postulate using the other postulates and axioms led to the discovery of several other geometries. Write True/False. Justify. [NCERT Exemplar]
23. Two salesmen make equal sale during the month of August. In September, each salesman doubles his sale of the month of August. Compare their sale in month of September.
24. Look at the given figure. Show that the length $AH >$ sum of the lengths $(AB + BC + CD)$.



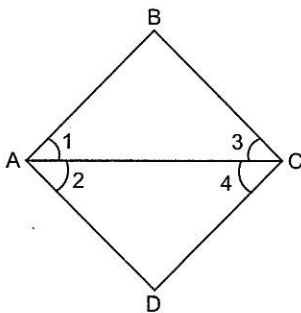
25. In the given figure, we have X and Y are the mid-points of AC and BC respectively and $AX = CY$. Show that $AC = BC$.



26. In the given figure, if $BX = \frac{1}{2}AB$, $BY = \frac{1}{2}BC$ and $AB = BC$, then show that $BX = BY$.

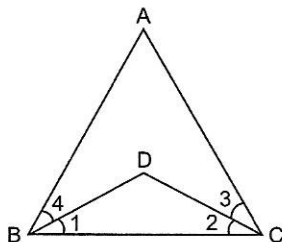


27. In the given figure, we have $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Show that $\angle A = \angle C$.



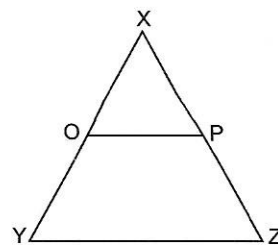
Short Answer Type Questions II [3 Marks]

28. In the given figure, we have $\angle ABC = \angle ACB$, $\angle 3 = \angle 4$. Show that $DB = DC$. [NCERT Exemplar]

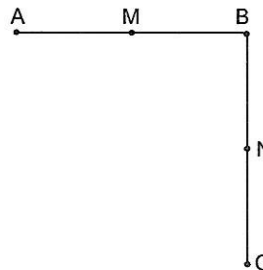


29. In the given figure, we have $AC = DC$ and $CB = CE$. Show that $AB = DE$.

30. In the given figure, if $OX = \frac{1}{2}XY$, $PX = \frac{1}{2}XZ$ and $OX = PX$. Show that $XY = XZ$. [CBSE 2016]



31. In the given figure,
 (i) $AB = BC$, M is mid-point of AB and N is mid-point of BC. Show that $AM = NC$
 (ii) $BM = BN$, M is mid-point of AB and N is mid-point of BC, Show that $AB = BC$.



[CBSE 2016]

Long Answer Type Questions [4 Marks]

32. Read the following statement:

An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all its angles are 60° each. Define the terms used in this definition which you feel necessary. Are there any undefined terms in this? Can you justify that all sides and all angles are equal in an equilateral triangle?

33. Study the following statement:

"Two intersecting lines cannot be perpendicular to the same line." Check whether it is an equivalent version to the Euclid's fifth postulate.

34. Read the following statements which are taken as axioms :

- If a transversal intersects two parallel lines, then corresponding angles are not necessarily equal.
- If a transversal intersect two parallel lines, then alternate interior angles are equal.

Is this system of axioms consistent? Justify your answer.

35. Read the following two statements which are taken as axioms:

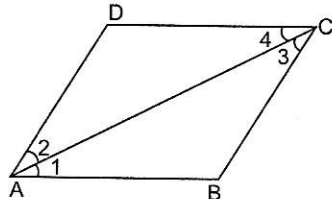
- If two lines intersect each other, then the vertically opposite angles are not equal.
- If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180° .

Is this system of axioms consistent? Justify your answer.



ASSESS YOURSELF

1. If a straight line falling on two straight lines makes the interior angles on the same side of it, whose sum is 135° , then two straight lines, if produced indefinitely, meet on the side on which the sum of angle is _____.
2. Jyoti is the same age as Ranjna. Kavita is also of the same age as Ranjna. State the Euclid's axiom that illustrates the relative age of Jyoti and Kavita.
- 3.




In the given quadrilateral ABCD, if

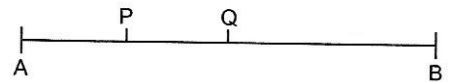
$$\angle 1 = \angle 3$$

$$\angle 2 = \angle 4 \text{ and}$$

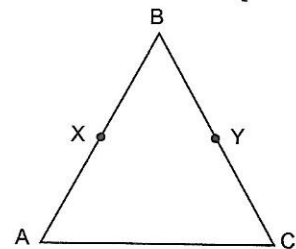
$$\angle 3 = \angle 4, \text{ then}$$

Find the relation between $\angle 1$ and $\angle 2$ by using Euclid's axiom.

4. Prove that two distinct lines cannot have more than one point in common.
5. State two equivalent versions of Euclid's fifth postulate.
6. Solve the equation $x + 10 = 30$ and state which axiom do you use here.
7. Consider the following statement, there exists a pair of straight lines that are everywhere equidistant from one another. Is this statement a direct consequence of Euclid's fifth postulate? Explain.
8.  In the above figure, if $AB = PQ$, $PQ = XY$, then $AB = XY$. State true or false. Justify your answer. [CBSE 2011]
9. If lines PQ, PR, PS and PT are parallel to a line l , what can be said about the points P, Q, R, S and T.
10. If Q is mid-point of line segment AB and P is mid-point of AQ then, show that $PQ = \frac{1}{4} AB$. [HOTS]



11. In the given figure, we have $AB = BC$, $BX = BY$. Show that $AX = CY$. [NCERT Exemplar]



12. Prove that an equilateral triangle can be constructed on any given line segment. [HOTS]
13. Read the following axioms:
 - (i) Things which are equal to the same thing are equal to one another.
 - (ii) If equals are added to equals, the whole is equal.
 - (iii) Things which are double of same thing are equal to one another.

Check whether the given system of axioms is consistent or inconsistent. [NCERT Exemplar]

14. "A square is a polygon made up of four line segments, out of which, length of three line segments are equal to the length of fourth one and all its angles are right angles".

Using Euclid's Axiom/postulates, justify that all angles and sides of a square are equal. [CBSE 2016]

15. Read the following statement.

"A rhombus is a polygon made up of four line segments, out of which, the length of three line segments are equal to the length of fourth one and its diagonal intersect at right angles."

Define the terms used in this definition which you feel necessary. Are there any undefined term in this? Can you justify that all sides of a rhombus are equal?